Integration

Indefinite Integral

2001 P2	6. Find $\int \frac{(x^2-2)(x^2+2)}{x^2}dx, \quad x \neq 0$	4
4. (JAN) 02 P1	Find $\int\left(\sqrt[3]{x} - \frac{1}{\sqrt{x}}\right)dx.$	4
2005 P2	1. Find $\int \frac{4x^3-1}{x^2}\,dx, \quad x \neq 0.$	4
2015 SP P1	1. Find $\int \frac{3x^3+1}{2x^2}\,dx, \; x \neq 0$.	4

Pre 2000 - Indefinite Integral

1	Find $\int\left(3x^3+4x\right)dx$.	3
2	Find $\int\left(2x^2+3\right)\,dx.$	3
3	Find $\int \frac{x^2-5}{x\sqrt{x}}\,dx.$	4

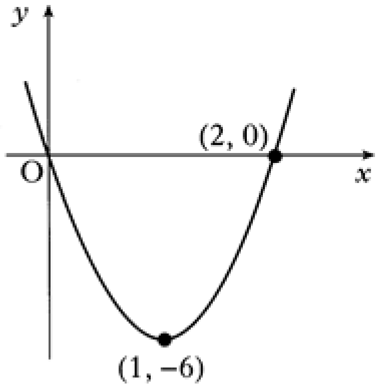
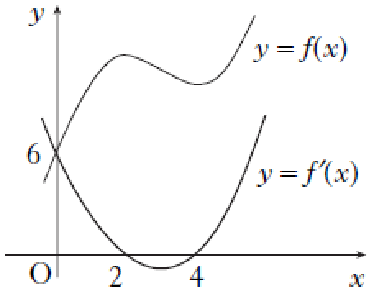
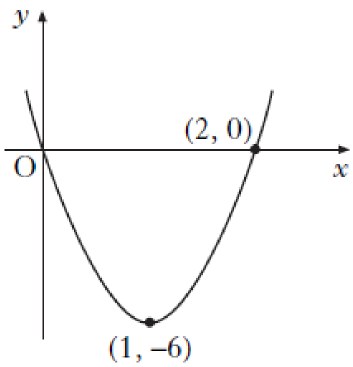
Definite Integral

2015 EP P1	3. Evaluate $\int_1^2 \frac{1}{6}x^{-2}\,dx$.	3

Pre 2000 - Definite Integral

1	Evaluate $\int_1^2\left(x^2+\frac{1}{x}\right)^2\,dx.$	5
2	Find the value of $\int_1^4 \sqrt{x}\,\,dx.$	4
3	Evaluate $\int_1^9 \frac{x+1}{\sqrt{x}}\,dx$	5
4	Find the value of $\int_1^2 \frac{u^2+2}{2u^2}\,du.$	5

Differential Equations

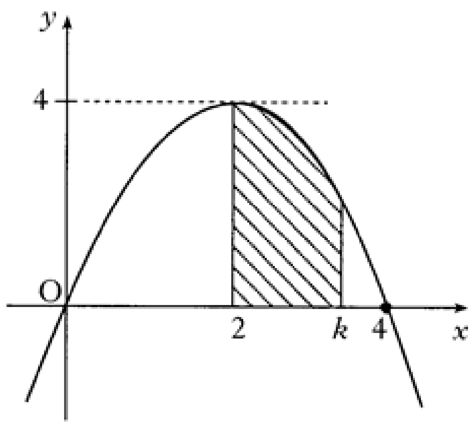
2004 P1	**11.** The diagram shows a parabola passing through the points (0, 0), (1, –6) and (2, 0). (*a*) The equation of the parabola is of the form $y = ax(x - b)$. Find the values of a and b. **3** (*b*) This parabola is the graph of $y = f'(x)$. Given that $f(1) = 4$, find the formula for $f(x)$. **5** 
2006 P2	**5.** The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point (–1, 9). Express y in terms of x. **4**
2007 P2	**10.** The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$. Both graphs pass through the point (0, 6). The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).  (*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$: (i) write down the values of a and b; (ii) find the value of k. **3** (*b*) Find the equation of the graph of the cubic function $y = f(x)$. **4**
2008 SP2 P1	**25.** The diagram shows a parabola with equation $y = 6x(x - 2)$. This parabola is the graph of $y = f'(x)$. Given that $f(1) = 4$, find the formula for $f(x)$. **5** 
2015 P1	**15.** The rate of change of the temperature, T °C of a mug of coffee is given by $\frac{dT}{dt} = \frac{1}{25}t - k$, $0 \leq t \leq 50$ • t is the elapsed time, in minutes, after the coffee is poured into the mug • k is a constant • initially, the temperature of the coffee is 100 °C • 10 minutes later the temperature has fallen to 82 °C. Express T in terms of t. **6**

2016 P2	**9.** For a function f, defined on a suitable domain, it is known that: • $f'(x) = \frac{2x+1}{\sqrt{x}}$ • $f(9) = 40$ Express $f(x)$ in terms of x.	**4**

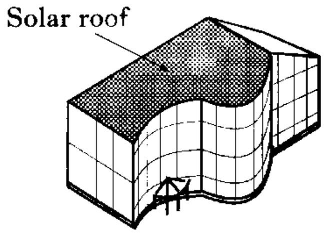
Pre 2000 - Differential Equations

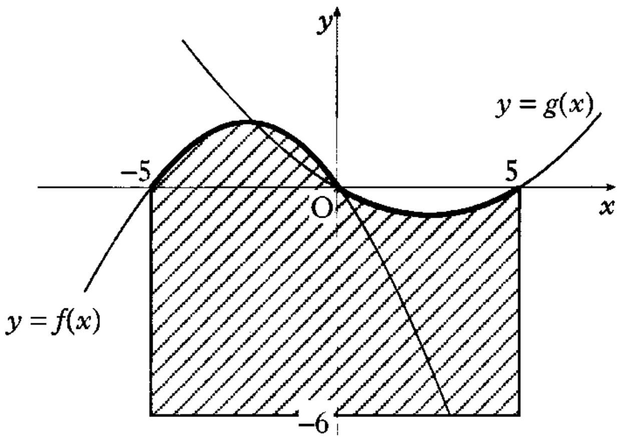
1	A curve, for which $\frac{dy}{dx} = 6x^2 - 2x$, passes through the point $(-1, 2)$. Express y in terms of x.	3
2	A curve for which $\frac{dy}{dx} = 3x^2 + 1$ passes through the point $(-1,2)$. Express y in terms of x.	4
3	A curve with equation $y = f(x)$ passes through the point $(2, -1)$ and is such that $f'(x) = 4x^3 - 1$. Express $f(x)$ in terms of x.	5
4	For all points on the curve $y = f(x)$, $f'(x) = 1 - 2x$. If the curve passes through the point $(2, 1)$, find the equation of the curve.	4
5	The graph of $y = g(x)$ passes through the point $(1,2)$. If $\frac{dy}{dx} = x^3 + \frac{1}{x^2} - \frac{1}{4}$, express y in terms of x.	4

Area under a curve

2000 P2	**A4.** The parabola shown crosses the x-axis at $(0, 0)$ and $(4, 0)$, and has a maximum at $(2, 4)$. The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$. (*a*) Find the equation of the parabola. (*b*) Hence show that the shaded area, A, is given by $A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$. 	**2** **3**

<table>
<tr>
<td>6.
(JAN)
02 P2</td>
<td>An energy efficient building is designed with solar cells covering the whole of its south facing roof. The energy generated by the solar cells is directly proportional to the area, in square units, of the solar roof.





The shape of the solar roof can be represented on the coordinate plane as the shaded area bounded by the functions $f(x)=\frac{1}{4}\left(-x^2-5x\right)$, $g(x)=\frac{1}{12}\left(x^2-5x\right)$ and the lines $x=-5$, $x=5$ and $y=-6$.

(a) Find the area of the solar roof.

(b) Ten square units of solar cells generate a maximum of 1 kilowatt.
What is the maximum energy the solar roof can generate in kilowatts (to the nearest kilowatt)?</td>
<td>

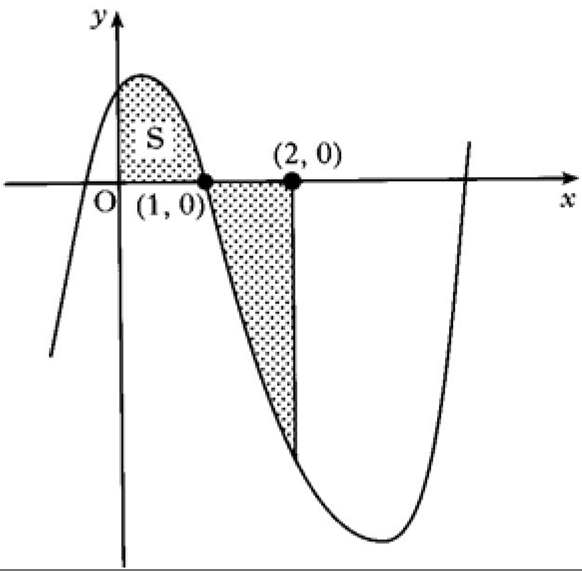
7

1</td>
</tr>
<tr>
<td>2006
P1</td>
<td>6. The graph shown has equation $y=x^3-6x^2+4x+1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x=2$.

(a) Calculate the shaded area labelled S.

(b) Hence find the total shaded area.

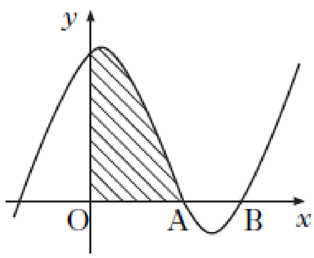

</td>
<td>4

3</td>
</tr>
<tr>
<td>2007
P1</td>
<td>8. The diagram shows a sketch of the graph of $y=x^3-4x^2+x+6$.

(a) Show that the graph cuts the x-axis at $(3, 0)$.

(b) Hence or otherwise find the coordinates of A.

(c) Find the shaded area.


</td>
<td>1

3

5</td>
</tr>
</table>

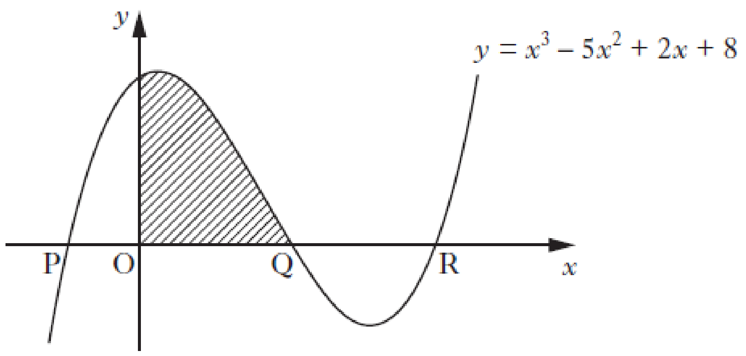
2012 P1

21. (*a*) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. **6**

(*b*) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x-axis at P, Q and R.

Determine the shaded area. **6**

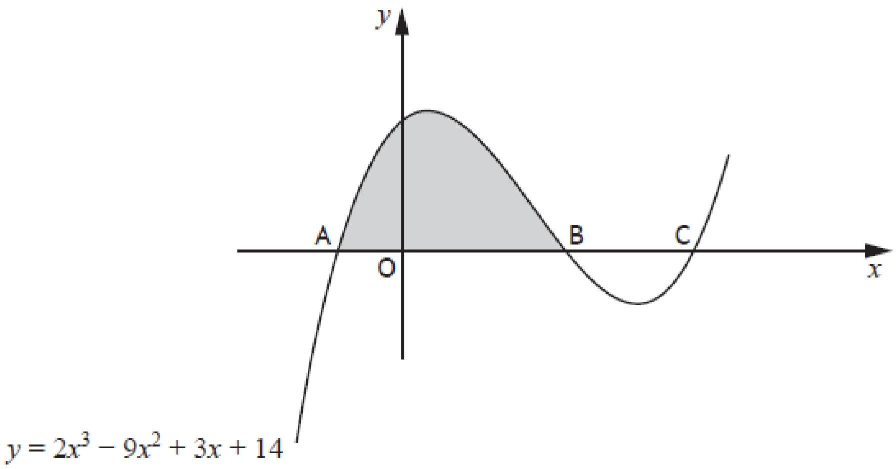
2016 P2

3. (a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. **2**

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. **3**

(b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$.

The curve cuts the x-axis at A, B and C.



(i) Write down the coordinates of the points A and B. **1**

(ii) Hence calculate the shaded area in the diagram. **4**

2017 P1

15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is $(7, 6)$.

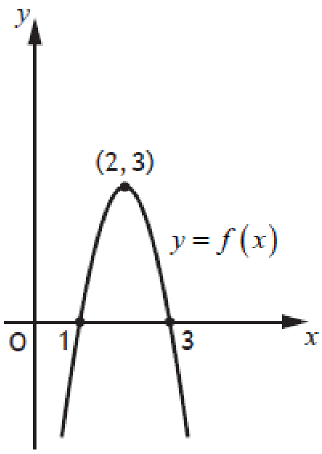


Diagram 1

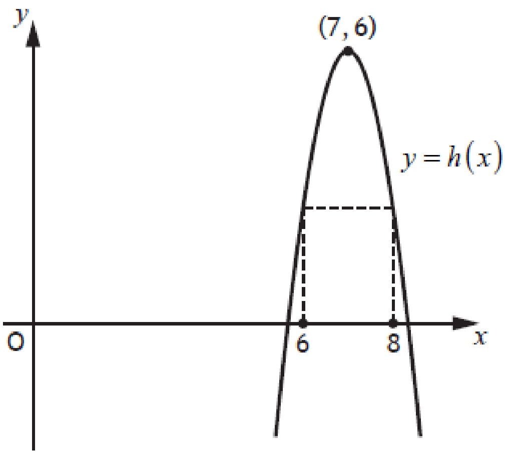


Diagram 2

(a) Given that $h(x) = f(x + a) + b$.

Write down the values of a and b. **2**

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

Determine the value of $\int_6^8 h(x)\,dx$. **1**

(c) Given $f'(1) = 6$, state the value of $h'(8)$. **1**

Pre 2000 - Area under a curve

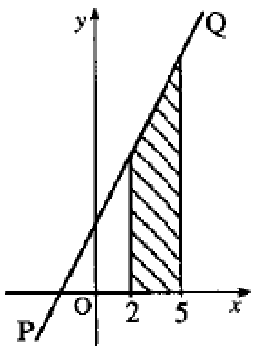
1

The line PQ has equation $y = 2x + 4$.

(a) Find, without using calculus, the area of the shaded trapezium shown in the diagram.

(b) Express the area of this trapezium as a definite integral.

(c) Evaluate this integral.



2, 1, 2

2

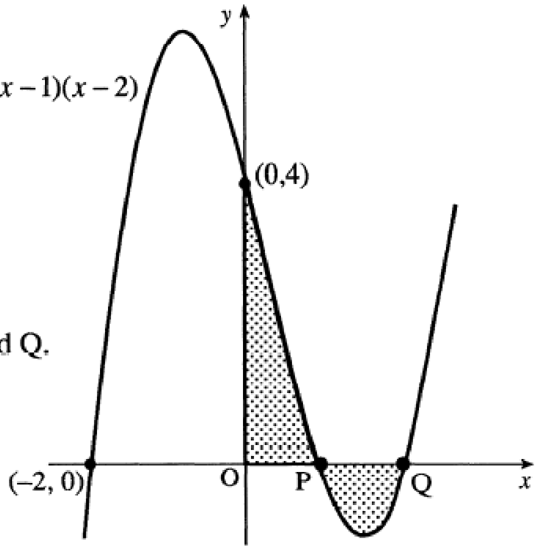
(a) Find the value $\int_1^2 \left(4 - x^2\right) dx$.

(b) Sketch a graph and shade the area represented by the integral in *(a)*.

3, 2

3 Evaluate $\int_1^2 \left(3x^2+4\right)\,dx$ and draw a sketch to illustrate the area represeneted by this integral. **5**

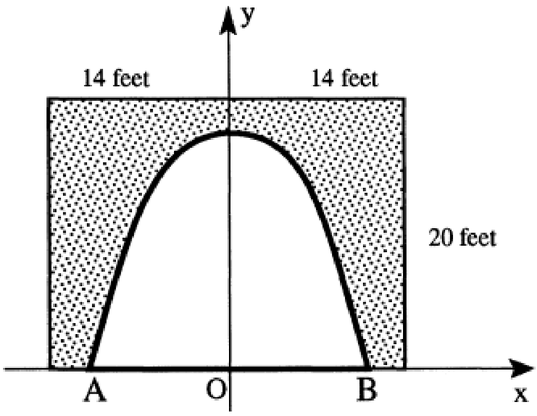
4 The diagram shows a sketch of the graph of $y=(x+2)(x-1)(x-2)$. The graph cuts the axes at $(-2, 0)$, $(0, 4)$ and the points P and Q.



(*a*) Write down the coordinates of P and Q. **(2)**

(*b*) Find the total shaded area. **(7)**

5 The concrete on the 20 feet by 28 feet rectangular facing of the entrance to an underground cavern is to be repainted.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 foot.The roof is in the form of a parabola with equation $y=18-\frac{1}{8}x^2$.

(*a*) Find the coordinates of the points A and B. **(2)**

(*b*) Calculate the total cost of repainting the facing at £3 per square foot. **(4)**

6 Functions f and g are defined on the set of real numbers by

$$f(x) = x - 1$$
$$g(x) = x^2$$

(*a*) Find formulae for (i) $f(g(x))$

(ii) $g(f(x))$.

(*b*) The function h is defined by $h(x) = f(g(x)) + g(f(x))$.

Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h.

(*c*) Find the area enclosed between this graph and the x-axis.

4, 3, 4

7 A function f is defined by the formula $f(x) = 4x^2(x-3)$ where $x \in \mathbf{R}$.

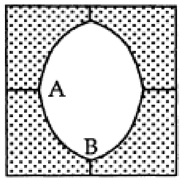
(*a*) Write down the coordinates of the points where the curve with equation $y = f(x)$ meets the x- and y-axes. **(2)**

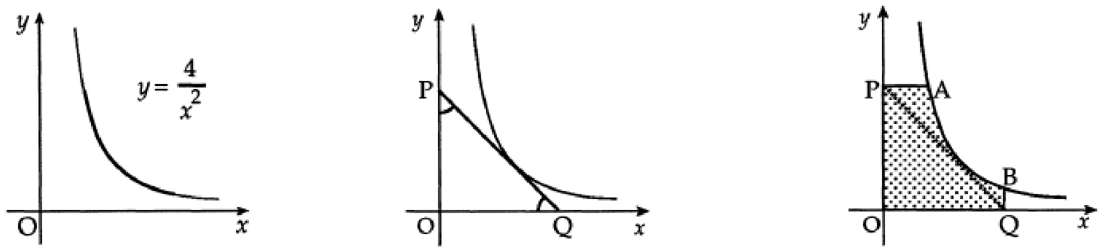
(*b*) Find the stationary points of $y = f(x)$ and determine the nature of each. **(6)**

(*c*) Sketch the curve $y = f(x)$. **(2)**

(*d*) Find the area completely enclosed by the curve $y = f(x)$ and the x-axis. **(4)**

8 The makers of "OLO", the square mint with the not-so-round hole, commissioned an advertising agency to prepare an illustration to the specification described in (i) to (iii) below. The finished illustration will look like the diagram on the right.





(i) The curve AB in the finished illustration is part of the curve with equation $y = \frac{4}{x^2}$.

(ii) A tangent to this curve, making equal angles with both axes, is to be drawn as shown (line PQ)

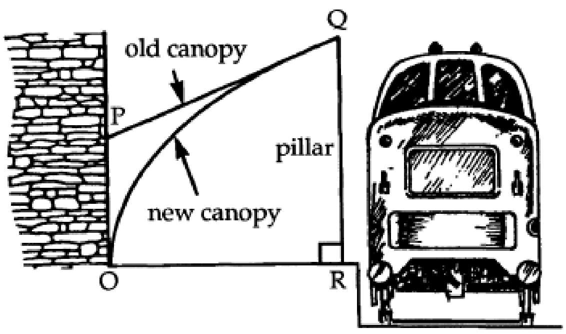
(iii) Straight lines perpendicular to the axes are to be drawn from P and Q as shown. The shaded part forms $\frac{1}{4}$ of the finished illustration.

(*a*) State the gradient of PQ and hence find the point of contact of the tangent PQ with the curve. **(5)**

(*b*) Find the equation of PQ and the coordinates of A and B. **(4)**

(*c*) Calculate the shaded area of the finished illustration. **(6)**

9 The diagram shows a proposed replacement of the old platform canopy at the local railway station by a new parabolic canopy, while keeping the original pillars.
If OR and OP are taken as the x- and y- axes and Q has coordinates (1, 1), then OQ has equation $y = \sqrt{x}$ and PQ is the tangent at Q to the parabola.



The planners have received an objection that there is a reduction of more than 10% in the space under the canopy and wish to compare the two canopies.

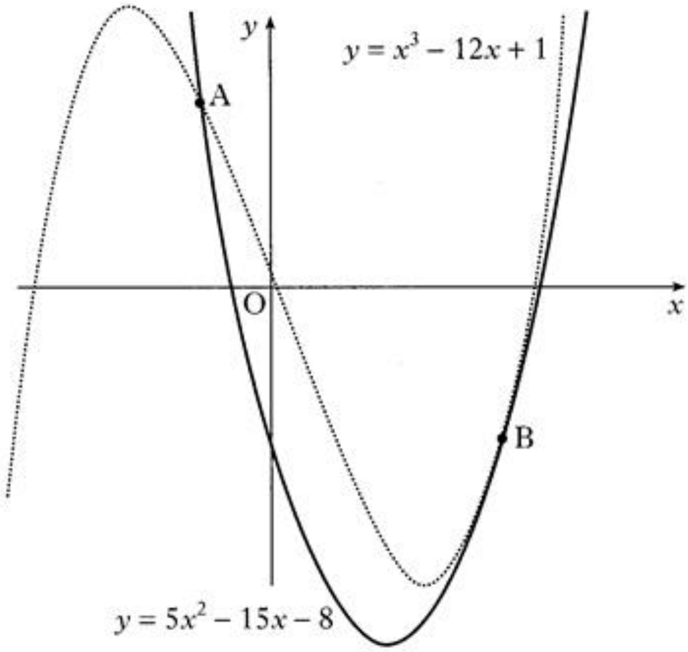
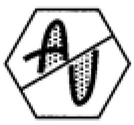
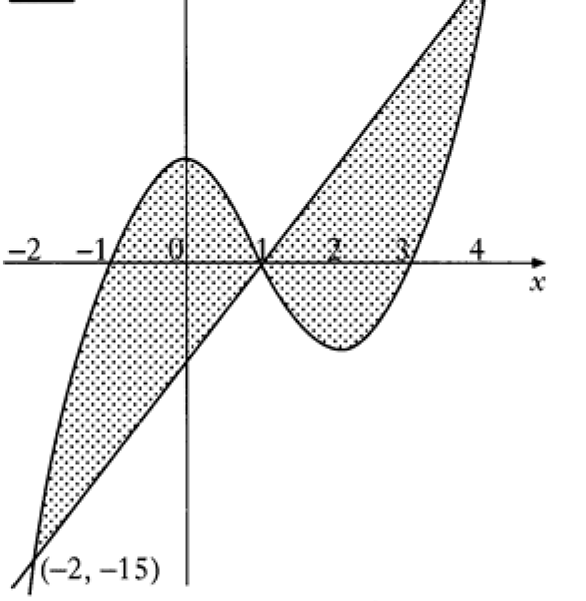
(*a*) Find the equation of the tangent PQ and the coordinates of P. **(5)**

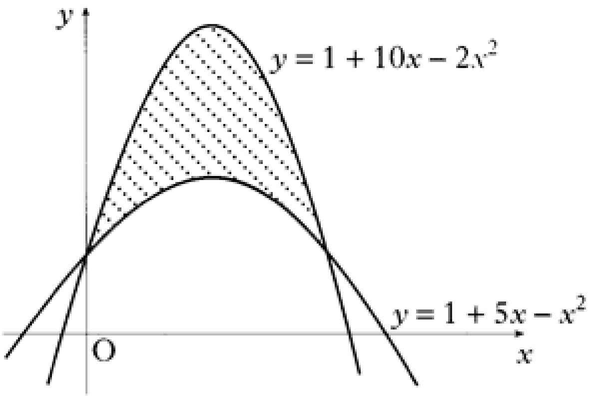
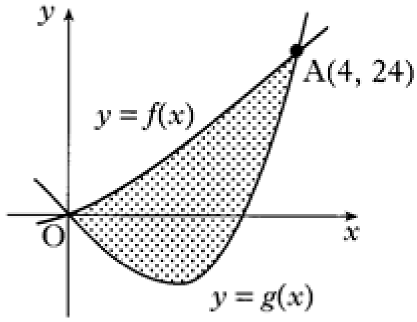
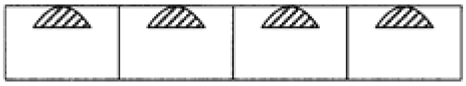
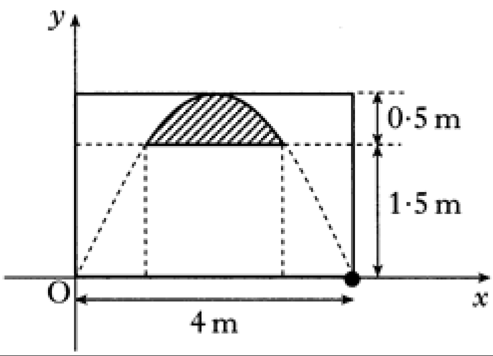
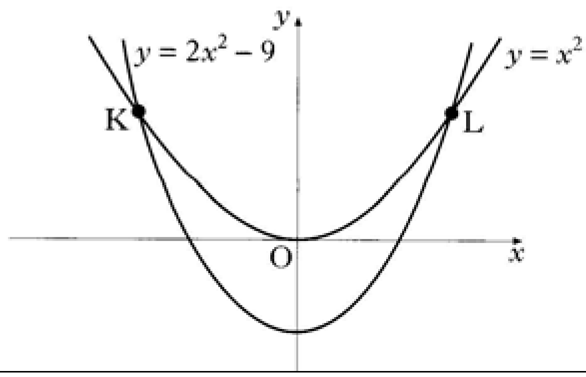
(*b*) Find the area of the trapezium OPQR. **(2)**

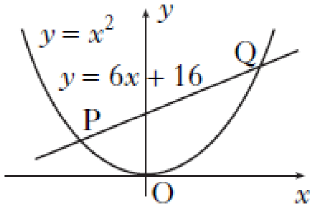
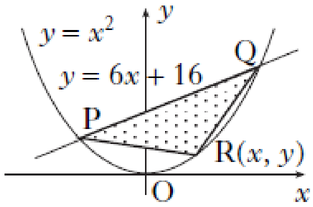
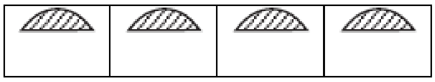
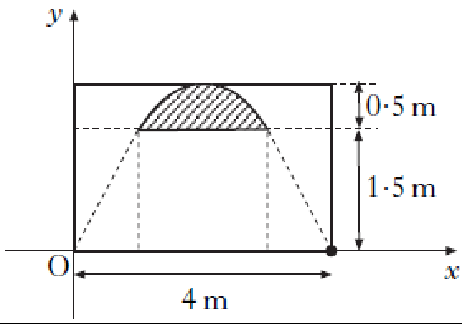
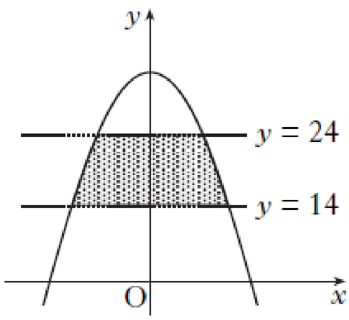
(*c*) Find the area under the parabola OQ. **(3)**

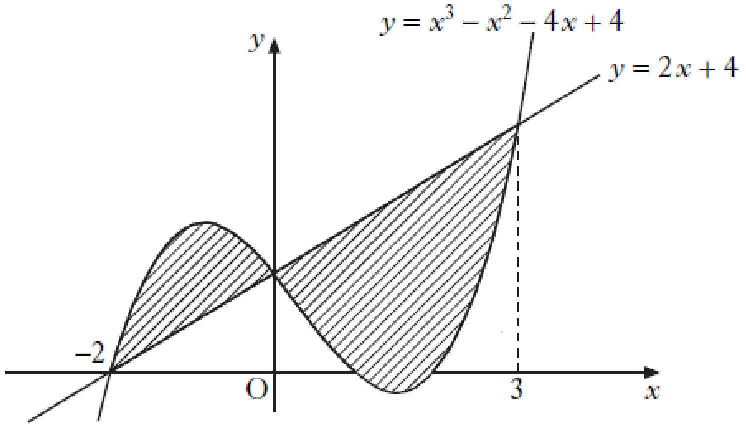
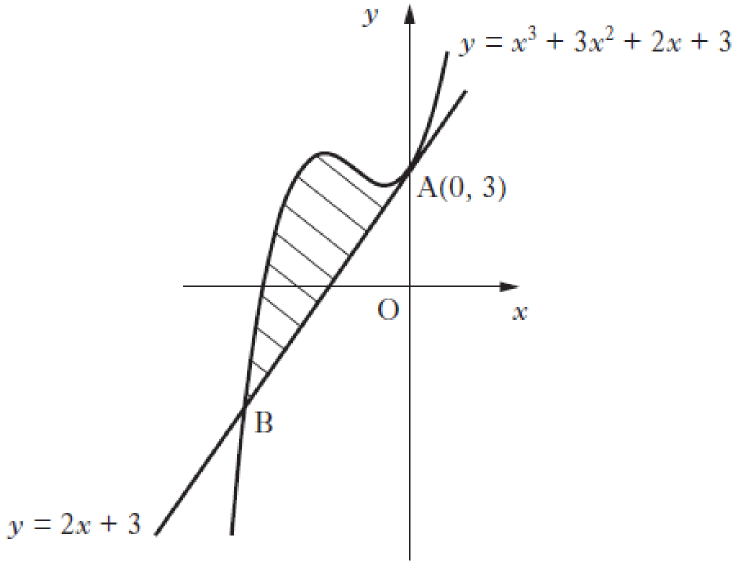
(*d*) Comment on the objection received. **(3)**

Area between 2 curves

2000 P1	**A4.** The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$. The two curves intersect at A and touch at B, ie at B the curves have a common tangent.  (*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. **4** (ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1** (*b*) The point A is $(-1, 12)$ and B is $(3, -8)$. Find the area enclosed between the two curves. **5**
2001 P2	**8.** A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.  A mathematical representation of the final logo is shown in the coordinate diagram.  The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry. Calculate the total shaded area. **7**

2002 P2	**5.** Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.  	**6**
2003 P2	**3.** The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin. Find the shaded area enclosed between the curves.  	**5**
2004 P2	**11.** An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.  The second diagram shows one such window. The shaded part represents the glass. The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$. Find the area in square metres of the glass in one window.  	**8**
2005 P2	**5.** The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown. Calculate the area enclosed between the curves.  	**8**

2008 SP1 P2	**5.** The diagram shows a curve with equation $y = x^2$ and a straight line with equation $y = 6x + 16$ intersecting the curve at P and Q. (*a*) Calculate the exact value of the area enclosed by the curve and the straight line.  The second diagram shows a third point, R, lying on the curve between P and Q. (*b*) The area, A, of triangle PQR, is given by $A(x) = -5x^2 + 30x + 80$. Determine the maximum area of this triangle, and express your answer as a fraction of the area enclosed by the curve and the straight line.  	7 4
2008 SP2 P2	**9.** An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.  The second diagram shows one such window. The shaded part represents the glass. The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$. Find the area in square metres of the glass in one window.  	8
2008 P2	**7.** The parabola shown in the diagram has equation $$y = 32 - 2x^2.$$ The shaded area lies between the lines $y = 14$ and $y = 24$. Calculate the shaded area.  	8

2011 P2	**4.** The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.  Calculate the total shaded area. **10**
2013 P2	**4.** The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.  The line meets the curve again at B. Show that B is the point $(-3, -3)$ and find the area enclosed by the line and the curve. **6**

2014 P2	**7.** Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents $300\,m^2$ of land.

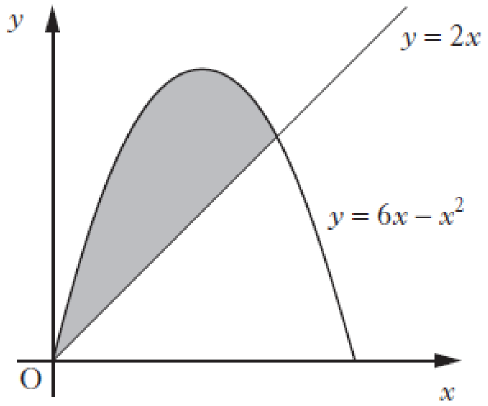


Diagram 1

(*a*) Calculate the area of land being developed. **5**

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

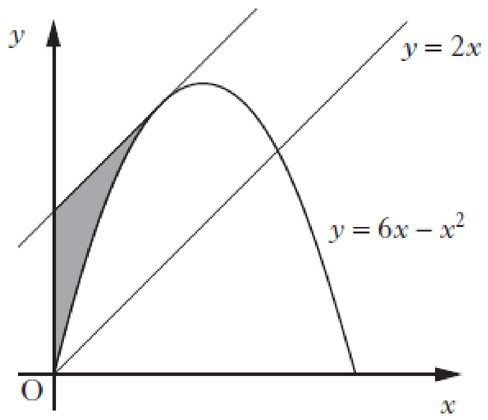
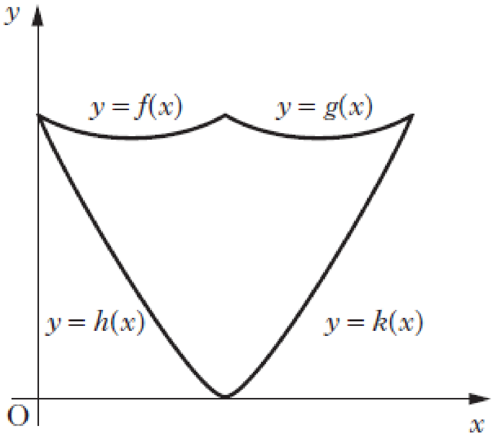
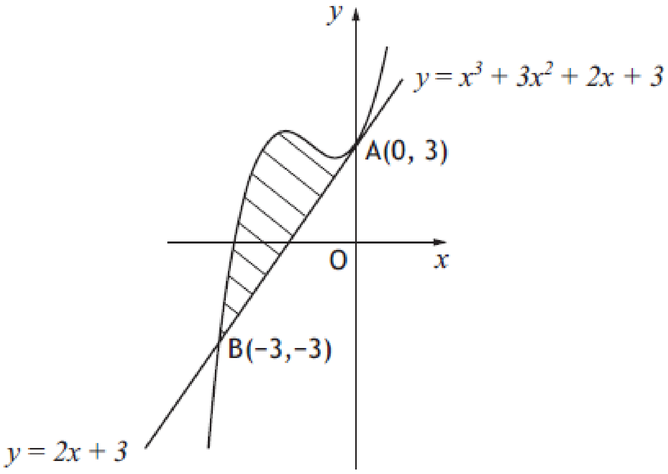


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

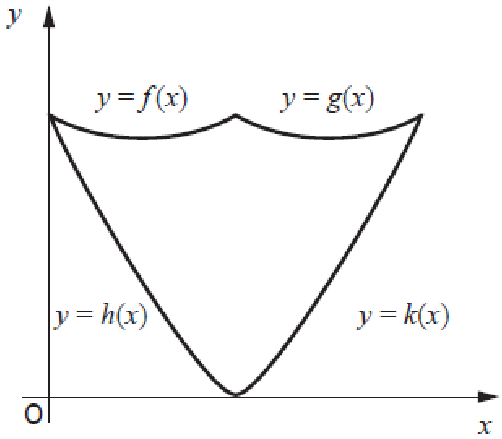
Calculate the area of the car park. **5**

2015 OLD P2	**4.** A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*". The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.  • $f(x)=\frac{1}{4}x^2-\frac{1}{2}x+3$ • $g(x)=\frac{1}{4}x^2-\frac{3}{2}x+5$ • $h(x)=\frac{3}{8}x^2-\frac{9}{4}x+3$ • $k(x)=\frac{3}{8}x^2-\frac{3}{4}x$ (*a*) Find the x-coordinate of the point of intersection of the graphs with equations $y=f(x)$ and $y=g(x)$. **2** The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis. The plaque has a vertical line of symmetry. (*b*) Calculate the area of the wall plaque. **7**

2015 EP P2	**4.** The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A (0, 3), as shown.  The line meets the curve again at B (−3,−3). Find the area enclosed by the line and the curve. **5**

2015 P2	**4.** A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*".

The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.



- $f(x)=\frac{1}{4}x^2-\frac{1}{2}x+3$
- $g(x)=\frac{1}{4}x^2-\frac{3}{2}x+5$
- $h(x)=\frac{3}{8}x^2-\frac{9}{4}x+3$
- $k(x)=\frac{3}{8}x^2-\frac{3}{4}x$

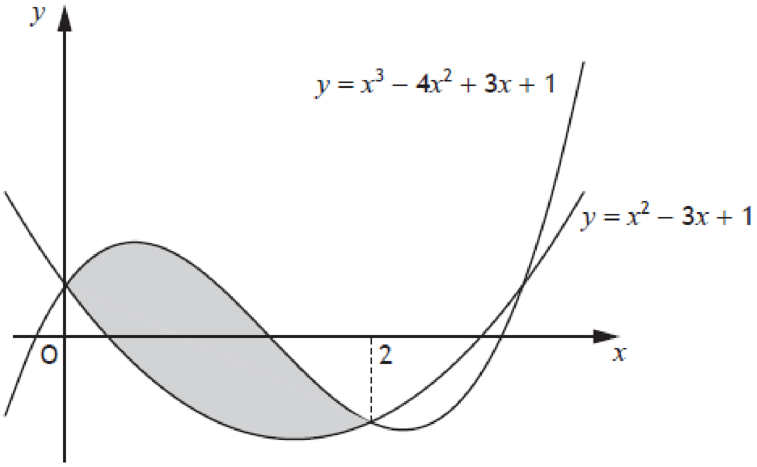
(a) Find the x-coordinate of the point of intersection of the graphs with equations $y=f(x)$ and $y=g(x)$. **2**

The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis.

The plaque has a vertical line of symmetry.

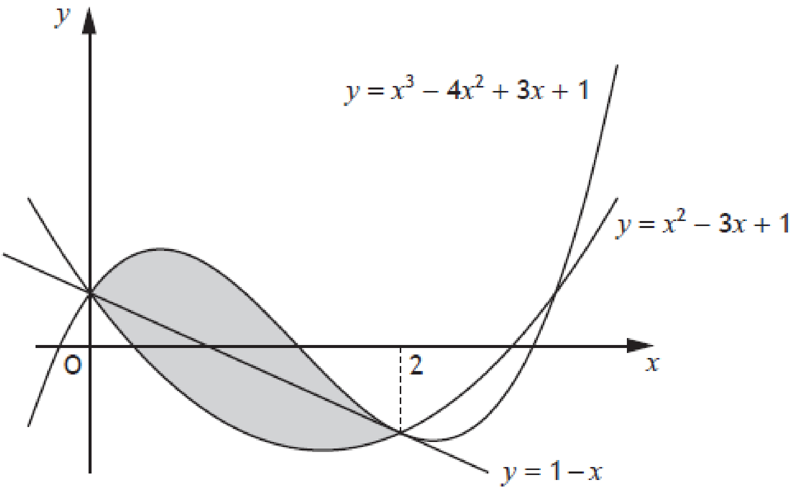
(b) Calculate the area of the wall plaque. **7**

2017 P1	**10.** Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.



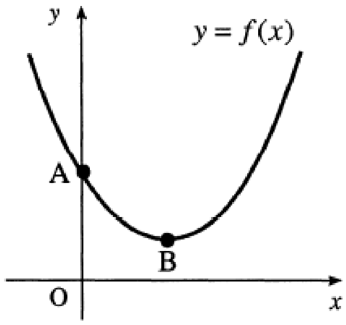
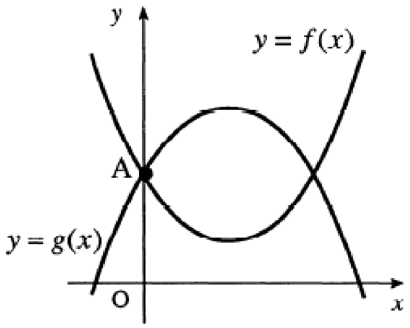
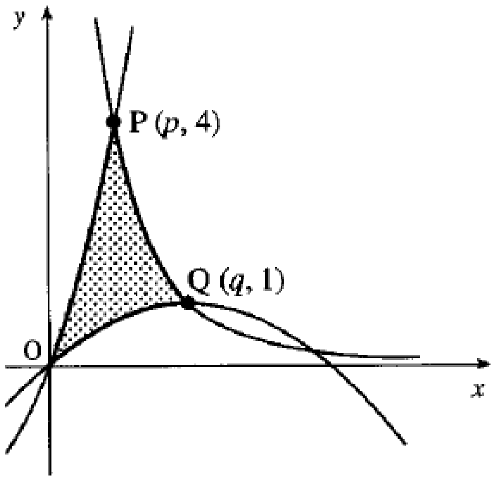
(a) Calculate the shaded area. **5**

The line passing through the points of intersection of the curves has equation $y = 1 - x$.



(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$. **4**

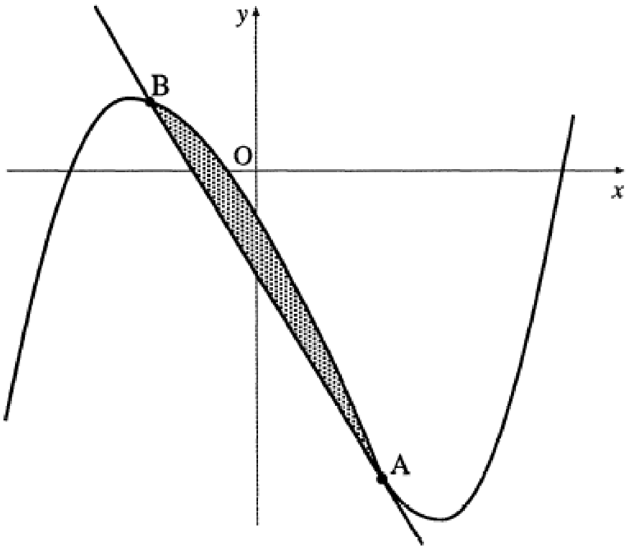
Pre 2000 - Area between 2 curves

1	(*a*) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. (*b*) Find the area completely enclosed between these two curves. **2, 3**
2	The first diagram shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x-2)^2 + 1$. The graph cuts the y-axis at A and has a minimum turning point at B.  (*a*) Write down the coordinates of A and B. **(3)**  (*b*) The second diagram shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$. Find the area enclosed by the two curves. **(5)** (*c*) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants. Write down the values of m and n. **(2)**
3	The origin, O, and the points P and Q are the vertices of a curved 'triangle' which is shaded in the diagram. The sides lie on curves with equations $y = x(x+3)$, $y = x - \frac{1}{4}x^2$ and $y = \frac{4}{x^2}$.  (*a*) P and Q have coordinates $(p, 4)$ and $(q, 1)$. Find the values of p and q. (*b*) Calculate the shaded area . **2, 7**

4 In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

(a) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(b) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



5 A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

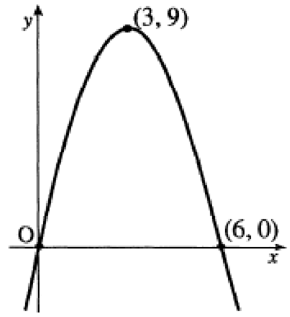


Diagram 1

(a) The parabola has equation of the form $y = ax(b - x)$. Determine the values of a and b. **(2)**

(b) Find the area enclosed by the parabola and the x-axis. **(4)**

(c) (i) Diagram 2 shows the parabola from (a) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

(ii) Calculate the area enclosed between the parabola and the line. **(5)**

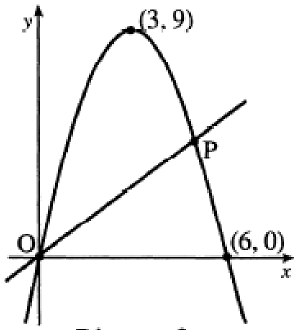
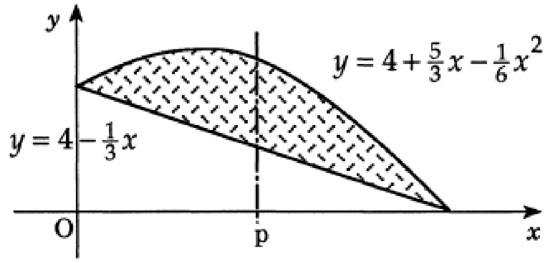


Diagram 2

6 When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.
The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.

(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**



(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.
By using your answer to part (*a*), or otherwise, show that

$$p^3 - 18p^2 + 432 = 0.$$ **(3)**

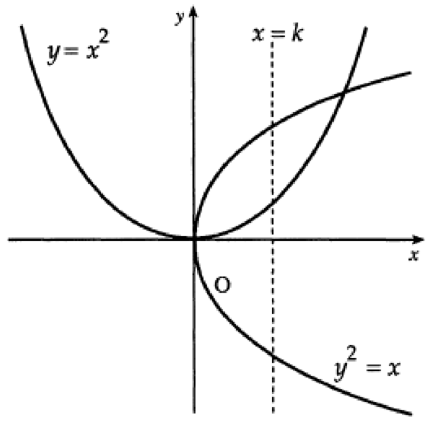
(*c*) (i) Verify that $p = 6$ is a solution of this equation.

(ii) Find algebraically the other two solutions of this equation.

(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

7 The diagram shows two curves with equations $y = x^2$ and $y^2 = x$.

The area completely enclosed between the two curves is divided in half by the line with equation $x = k$.

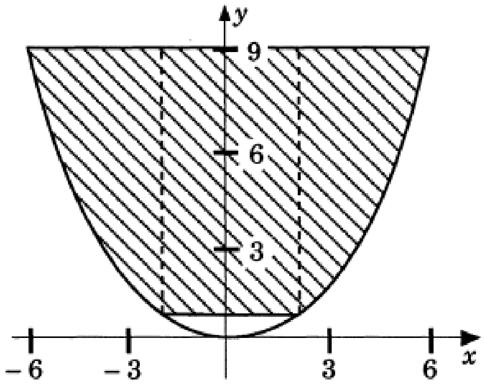


(*a*) Represent these two equal areas by two separate integrals each involving k. **(6)**

(*b*) Equate the integrals and show that k is given by the equation

$$2k^3 - 4k^{\frac{3}{2}} + 1 = 0.$$ **(4)**

(*c*) Use the substitution p^2 for k^3 to find the value of k. **(4)**

8	The cargo space of a small bulk carrier is 60m long.  The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y=\frac{1}{4}x^2$, $-6 \le x \le 6$, between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry.  **(9)**